Multiple View Geometry: Solution Sheet 7

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Exercise: June 18th, 2020

Part I: Theory

1. (a) l is coimage of L, and therefore l is normal vector to the plane that is determined by the camera position and L.

$$\Rightarrow \begin{matrix} l^T x_1 = 0 \\ l^T x_2 = 0. \end{matrix}$$

$$\Rightarrow l \sim x_1 \times x_2 = \hat{x_1} x_2.$$

l_1 and l_2 are normal vectors to the planes through camera position and L_1, L_2 respectively.

$$\Rightarrow \begin{matrix} l_1^T x = 0 \\ l_2^T x = 0 \end{matrix}$$

$$\Rightarrow x \sim l_1 \times l_2 = \hat{l_1} l_2.$$

   (b)   i. $l_1 \sim \hat{x} u$:
          x is in the preimage of L_1. $\Rightarrow l_1^\top x = 0$.
          $\exists$ point $u \neq p$ in L_1. $\Rightarrow l_1^\top u = 0$
          $\Rightarrow l_1 \sim \hat{x} u$.
        ii. $l_2 \sim \hat{x} v$: analog to i.
        iii. $x_1 \sim \hat{l} r$:
          x_1 is in the preimage of L. $\Rightarrow x_1^\top l = 0$
          $\exists$ a line L' through p_1 with coimage $r \neq l$. $\Rightarrow x_1^\top r = 0$.
          $\Rightarrow x_1 \sim \hat{l} r$.
        iv. $x_2 \sim \hat{l} s$: analog to iii.

2. $\mathrm{rank} \begin{pmatrix} \hat{x_1} \Pi_1 \\ \hat{x_2} \Pi_2 \end{pmatrix} \leqq 3$

$\Rightarrow \exists X \in \mathbb{R}^4 \backslash \{0\}$ with $\begin{pmatrix} \hat{x_1} \Pi_1 \\ \hat{x_2} \Pi_2 \end{pmatrix} X = 0$.

$\Rightarrow \hat{x_1} \Pi_1 X = 0 \ \wedge \ \hat{x_2} \Pi_2 X = 0,$

$\Rightarrow x_1 \times \Pi_1 X = 0 \ \wedge \ x_2 \times \Pi_2 X = 0.$

$\Rightarrow x_1$ and $\Pi_1 X$ are linearly dependent; and x_2 and $\Pi_2 X$ are linearly dependent.

$\Rightarrow \exists \lambda_1, \lambda_2 \in \mathbb{R}$ with $\Pi_1 X = \lambda_1 x_1 \ \wedge \ \Pi_2 X = \lambda_2 x_2$

$\Rightarrow x_1$ and x_2 are projections of X.

3. $\exists \lambda \in \mathbb{R} : [R', T'] = \lambda [R, T] H = \lambda [R, T] \begin{bmatrix} I & 0 \\ v^\top & v_4 \end{bmatrix} = \lambda [R + Tv^\top, Tv_4]$

$$
\begin{aligned}
E' &= \hat{T}'R' \\
&= (\widehat{\lambda v_4 T}) \cdot (\lambda(R + Tv^\top)) \\
&= \lambda^2 v_4 \hat{T}(R + Tv^\top) \\
&= \lambda^2 v_4 \hat{T}R + \lambda^2 v_4 \underbrace{\hat{T}T}_{=0} v^\top \\
&= \lambda^2 v_4 \hat{T}R \\
&= \lambda^2 v_4 E \quad \text{with } \lambda^2 v_4 \in \mathbb{R}
\end{aligned}
$$

$\Rightarrow \; E' \sim E$