

Weekly Exercises 8

Room: 02.09.023

Wednesday, 10.07.2019, 12:15 - 14:00

MAP infer and BN learn (Due: 08.07) (14+6 Points)

Exercise 1 (8 Points). (Graphcut) Consider a simple MRF with 2 connected binary nodes and the following energies:

$$E_1 = \begin{bmatrix} 5 \\ 6 \end{bmatrix}, E_2 = \begin{bmatrix} 8 \\ 7 \end{bmatrix}, E_{1,2} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \quad (1)$$

- 1) Construct new energies $\tilde{E}_1, \tilde{E}_2, \tilde{E}_{1,2}$ that can be used for flow graph while keeping the underlying joint distribution invariant.
- 2) Draw the flow graph and the minimum cut that you have found and conclude with the labeling of the labels that minimized the total energy.

Exercise 2 (6 Points). (MLE of BN) In the lecture we have seen that for fully observed Bayesian network, the MLE estimate of tabular CPD parameters has a close form of ratio of counts $\frac{\#(x_{F_i})}{\#(x_{Pa(i)})}$, where $F_i = \{i\} \cup Pa(i)$. You are asked to derive this result from the general problem setting as follows:

$$\operatorname{argmin}_{\theta \in \mathbb{R}_+^{|\theta|}} - \frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{V}} \sum_{x' \in \mathcal{D}} \sum_{x_{F_i}} \log \theta(x_i | x_{Pa(i)}) \mathbf{1}\{x_{F_i} = x'_{F_i}\} \quad (2)$$

$$\text{s.t. } \forall (i, x), \sum_{x_i} \theta(x_i | x_{pa(i)}) = 1 \quad (3)$$

Hint: introducing Lagrangian will help solving this problem.

Exercise 3 (6 Points). (Semiring) Interestingly, sum-product and max-sum operators are deeply linked with the semiring algebraic structure. Look up the mathematical definition of semiring on e.g. Wikipedia, then verify that $([0, +\infty), +, \times)$, $([0, +\infty), \max, \times)$ and $([-\infty, +\infty), \max, +)$ are all semirings.