Probabilistic Graphical Models in Computer Vision

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Weekly Exercises 7

Room: 02.09.023

Wednesday, 03.07.2019, 12:15 - 14:00

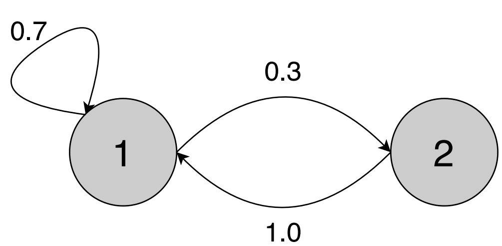
MCMC sampling (Due: 01.07) (14+4 Points)

Exercise 1 (4 Points)**.** (regular $\Rightarrow$ irreducible and aperiodic) Show that if a Markov chain is regular, then it is irreducible and aperiodic.

Exercise 2 (8 Points)**.** (Properties of transition matrix) For Markov chains with finite state spaces, the transition kernel can be represented in matrix form: $T_{l,m} = Pr(X_i = m | X_{i-1} = l)$ for any time step i. For each of the transition matrices in the following, determine whether it is irreducible / aperiodic:

$$\underset{\text{(a)}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \quad \underset{\text{(b)}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \quad \underset{\text{(c)}}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} \quad \underset{\text{(d)}}{\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}}$$

Exercise 3 (6 Points)**.** (State transition diagram and stationary distribution) A transition matrix of a finite state Markov chain can be visualized as a diagram with nodes representing the states and directed arcs the possibles transitions and its probability. For example, a diagram for the transition matrix $\begin{bmatrix} 0.7 & 0.3 \\ 1.0 & 0.0 \end{bmatrix}$ looks like the following:



For each of the transition matrices (a), (c), (d) in Exercise 2, draw a corresponding diagram, write down the set of stationary distributions, and for each stationary distribution indicate the set of initial distributions that will converge towards it.

Programming (Due:08.07) (12 Points)

In this programming exercise, you are asked to use mean field and Gibbs sampling for image segmentation. See the ipython file for more details.