Probabilistic Graphical Models in Computer Vision

Lecture: Dr. Tao Wu
Exercises: Yuesong Shen, Zhenzhang Ye
Summer Semester 2019

Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 6

Room: 02.09.023

Wednesday, 26.06.2019, 12:15 - 14:00

Sampling and MCMC (Due: 24.06) (8+4 Points)

Exercise 1 (4 Points)**.** Given a random number generator following the uniform distribution $\mathcal{U}(0,1)$. How to generate samples from distribution $\mathcal{U}(a,b)$ for some $a < b$? Justify your answer.

Exercise 2 (8 Points)**.** Derive the polar form of Box-Muller method: Let $(X_1, X_2) \sim \mathcal{N}(0, I_2)$, let $(Z_1, Z_2) \sim \frac{1}{\pi}\mathbf{1}\{z_1^2 + z_2^2 < 1\}$,

1) Let (T, γ) be the polar transform of (Z_1, Z_2), show that we have $T^2 \sim \mathcal{U}(0,1)$, $\gamma \sim \mathcal{U}(0, 2\pi)$ and that T, γ are independent;

2) Let (R, θ) be the polar transform of (X_1, X_2), show that we have $R^2 \sim \exp(1/2)$, $\theta \sim \mathcal{U}(0, 2\pi)$ and that R, θ are independent;

3) Using 1) and 2), show that by setting $R^2 = -2\log T^2, \theta = \gamma$, R, θ satisfy the required form in 2) and we can derive the following sample transformation:

$$x_i = z_i\sqrt{\frac{-2\log(z_1^2 + z_2^2)}{z_1^2 + z_2^2}}, i \in \{1, 2\} \tag{1}$$