Probabilistic Graphical Models in Computer Vision

Lecture: Dr. Tao Wu
Exercises: Yuesong Shen, Zhenzhang Ye
Summer Semester 2019

Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 5

Room: 02.09.023

Wednesday, 19.06.2019, 12:15 - 14:00

Junction Tree Algorithm (Due:17.06) (12+6 Points)

Exercise 1 (4 Points)**.** Given a pairwise MRF $\mathcal{H} = (\mathcal{V}, \mathcal{E})$,

1. show that the messages between two variable nodes in Belief Propagation can be simplified as:

$$m_{i\to j}(y_j) = \log \sum_{y_i} \exp\left(-E_i(y_i) - E_{ij}(y_i, y_j) + \sum_{k\in\mathcal{N}_{\mathcal{H}}(i)\setminus\{j\}} m_{k\to i}(y_i)\right). \quad (1)$$

   Hint:Firstly, consider a leaf node in the tree. Then assume the messge on a node satisfies the equation, show that the message to its parent satisfies as well.

2. show that the node marginal can be computed by:

$$\mu_i(y_i) = \exp\left(-E_i(y_i) + \sum_{k\in\mathcal{N}_{\mathcal{H}}(i)} m_{k\to i}(y_i) - \log Z\right). \quad (2)$$

Exercise 2 (4 Points)**.** Considering the same graph in sheet 4 as shown in Figure 1 and assuming we know $\phi(X_1, X_2, X_3)$, here we use Belief Propagation to compute $p(x_5)$:

1. Draw the corresponding factor graph. Try to keep the tree structure in factor graph.

2. Apply Belief Propagation to compute $p(x_5)$. Write down the messages passed in whole graph and the final formula to compute $p(x_5)$.

Exercise 3 (6 Points)**.** Consider the bayesian network in Figure 2:

1. Moralize and triangulate it. Draw the graph.

2. Draw the junction tree with order x_1, x_2, x_8, x_3, x_7.

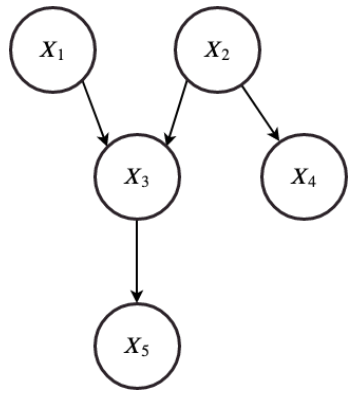

Figure 1: Exercise 2

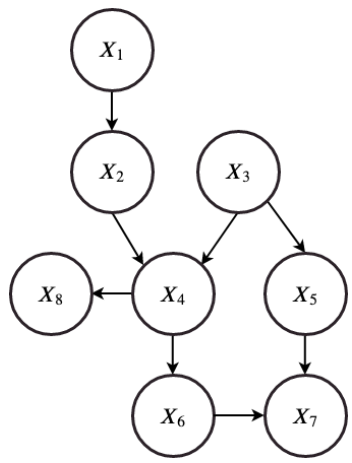

Figure 2: Exercise 3

Exercise 4 (4 Points)**.** Given two distributions q, p, show the following properties of KL:

$$\mathrm{KL}(q|p) \geq 0, \forall q, p$$

Hint: First show that $\log x \leq x - 1$ and you can assume we use the natural logarithm here.

Programming(Due:24.06) (12 Points)

In this programming exercise, you are asked to implement belief propagation. See the ipython file for more details.