



Multiple View Geometry: Exercise Sheet 10

Prof. Dr. Daniel Cremers, David Schubert, Mohammed Brahim
Computer Vision Group, TU Munich

<http://vision.in.tum.de/teaching/ss2019/mvg2019>

Exercise: July 17th, 2019

Part I: Theory

The following exercises should be **solved at home**. Writing down your answer will help you present and discuss the answer during the tutorial.

1. Variational Calculus and Euler-Lagrange

Let $u: \mathbb{R}^n \rightarrow \mathbb{R}$ be a smooth scalar function and $E(u)$ an energy functional given by

$$E(u) = \int_{\Omega} \mathcal{L}(u(x), \nabla u(x)) \, dx .$$

The Gâteaux derivative of E at u in direction $h: \mathbb{R}^n \rightarrow \mathbb{R}$ is given by

$$\left. \frac{dE(u)}{du} \right|_h := \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} [E(u + \epsilon h) - E(u)] = \int_{\Omega} \frac{dE(u)}{du}(x) \cdot h(x) \, dx .$$

(a) Under the assumption that h vanishes at the boundary of Ω , prove that

$$\frac{dE(u)}{du} = \frac{\partial \mathcal{L}(u, \nabla u)}{\partial u} - \operatorname{div} \left(\frac{\partial \mathcal{L}(u, \nabla u)}{\partial (\nabla u)} \right) .$$

(b) Which condition must hold true for a minimizer u_0 of $E(u)$...

- ... in general?
- ... if $\mathcal{L}(u, \nabla u) = \mathcal{L}(u)$?
- ... if $\mathcal{L}(u, \nabla u) = \mathcal{L}(\nabla u)$?

2. Multiview Reconstruction as Shape Optimization

You saw in the lecture that 3D reconstruction from multiple views can be posed as a variational problem. Let $\rho: V \rightarrow [0, 1]$ be the photoconsistency function, and $u: V \rightarrow \{0, 1\}$ the indicator function of the object to be reconstructed. We want to minimize (see Chapter 10, slide 10)

$$E(u) = \int_V \rho(x) |\nabla u(x)| \, dx$$

under the constraints

$$\begin{cases} \int_{R_{ij}} u(x) \, dR_{ij} \geq 1 & \text{if } j \in S_i , \\ \int_{R_{ij}} u(x) \, dR_{ij} = 0 & \text{else .} \end{cases}$$

(a) Write down the Euler-Lagrange equation for the given energy $E(u)$.

Gradient descent for energy functionals is performed in analogy to gradient descent on multivariate functions: from an estimate $u^{(k)}(x)$, the estimate in iteration $k + 1$ is obtained by going in negative gradient direction:

$$u^{(k+1)} = u^{(k)} - \tau \frac{dE(u)}{du}$$

with step size τ . This is a discretization of the differential equation from the lecture.

(b) Write down one gradient descent iteration for $E(u)$.

Part II: Practical Exercises

Finish the practical exercises from previous sheets.