Chapter 0
Organization and Overview

Convex Optimization for Machine Learning & Computer Vision
SS 2018

Tao Wu
Emanuel Laude
Zhenzhang Ye

Computer Vision Group
Department of Informatics
TU Munich

Organization and Overview

**Tao Wu
Emanuel Laude
Zhenzhang Ye**

Organization

Course Overview

Organization

Whether this lecture fits you?

Prerequisites

- Background in mathematical analysis and linear algebra

- Numerical implementation in Matlab or Python

- Interest in mathematical theory (know why!)

Organization and Overview

Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization

Course Overview

Whether this lecture fits you?

Prerequisites

- Background in mathematical analysis and linear algebra

- Numerical implementation in Matlab or Python

- Interest in mathematical theory (know why!)

Nice plus (but not necessary)

- Experience in machine learning or computer vision
  e.g., CV I & II, ML for CV, Probab. Graphical Models in CV.

- Knowledge in continuous optimization
  e.g., Nonlinear Optimization.

- Knowledge in functional analysis
  Further understanding of the theory in full generality.

Course overview

Lectures

1. Theory of convex analysis

2. Design and analysis of optimization algorithms

3. Selected topic: Stochastic optimization

Applications are mostly covered in exercise session...

Exercise session

Organizers: Emanuel Laude and Zhenzhang Ye

- Exercise sheets covering the content of the lecture will be passed out every Wednesday.
- Exercises contain theoretical as well as programming questions.
- If solutions have obviously been copied, both groups will get 0 points.
- You may work on the exercises in groups of two.
- You are encouraged to present your solution on board at exercise class.
- To get a 0.3 grade bonus, you need to complete 75% of the total exercise points.
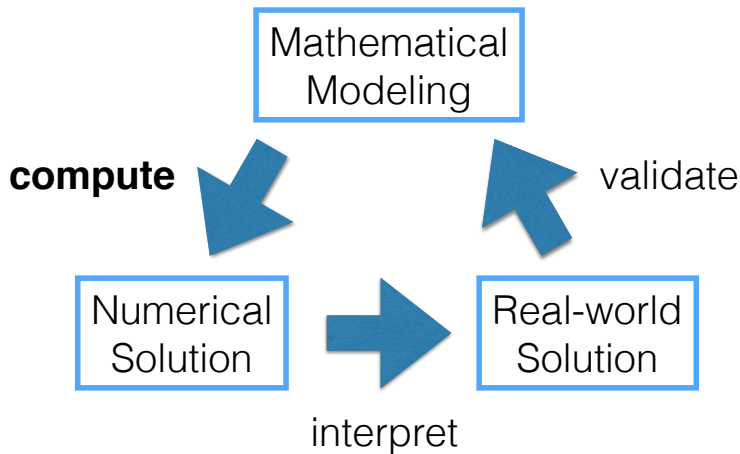
Contact us

Miscellaneous info

- Tao's office: 02.09.061
- Emanuel's office: 02.09.039
- Zhenzhang's office: 02.09.037
- Office hours: Please write an email.
- Lecture: Starts at quarter past. Short break in between.
- Course website:
  https://vision.in.tum.de/teaching/ss2018/cvx4cv

First Glimpse of the Course

Driving cycle

Organization and Overview

Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization
Course Overview

Mathematical Modeling

compute

validate

Numerical Solution

Real-world Solution

interpret

An example from computer vision / machine learning

Organization and Overview

Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization

Course Overview

- Image segmentation / multi-labeling:

image   segmentation ($L = 4$)

An example from computer vision / machine learning

- Image segmentation / multi-labeling:

<div>

image

segmentation ($L = 4$)

</div>

- Variational method for finding label function $u : \Omega \to \Delta^L$

$$\text{minimize} \sum_{j \in \Omega} \left(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j \rangle \right) + \alpha \sum_{l=1}^{L} \sum_{i} \omega_i \|(\nabla u^l)_i\|,$$

where

- Pointwise constraint: Δ^L is the unit $(L-1)$-simplex.
- Unary term: $f : \Omega \to \mathbb{R}^L$ is a pre-computed vector.
- Pairwise term: $\sum_i \omega_i \cdot (\nabla u^l)_i$ is the weighted total-variation.

Prototypical workflow

Organization and Overview

Tao Wu
Emanuel Laude
Zhenzhang Ye

Organization

Course Overview

- The variational model

$$\text{minimize} \sum_{j \in \Omega} \Big(\delta\{u_j \in \Delta^L\} + \langle u_j, f_j \rangle \Big) + \alpha \sum_{l=1}^{L} \sum_i \omega_i \|(\nabla u^l)_i\|,$$

  is a special case of **convex optimization**

$$\text{minimize } J(u) + \delta\{u \in C\},$$

  with **convex objective** J and **convex constraint** C.

- This course is about **theory** and **practice** for solving convex optimization (arising from computer vision and machine learning).

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

where F, G are convex functions, K is a linear operator.

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

  where F, G are convex functions, K is a linear operator.

- Reformulate the problem (by introducing *dual variable p*):

$$\max_p \min_u \langle Ku, p \rangle - F^*(p) + G(u),$$

  where F^* is the *convex conjugate* of F.

Prototypical workflow

- Put into canonical form:

$$\min_u F(Ku) + G(u),$$

  where F, G are convex functions, K is a linear operator.

- Reformulate the problem (by introducing *dual variable p*):

$$\max_p \min_u \langle Ku, p \rangle - F^*(p) + G(u),$$
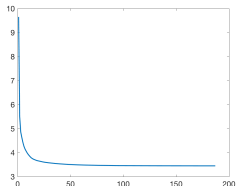
  where F^* is the *convex conjugate* of F.

- Apply an iterative scheme (e.g. PDHG):

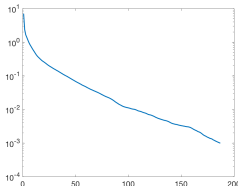$$u^{k+1} = \arg\min_u \left\langle u, K^\top p^k \right\rangle + G(u) + \frac{s}{2}\|u - u^k\|^2,$$

$$p^{k+1} = \arg\min_p - \left\langle K(2u^{k+1} - u^k), p \right\rangle + F^*(p) + \frac{t}{2}\|p - p^k\|^2.$$

Questions of our concern

energy value

primal-dual gap

- Does a minimizer always exist?
- How to characterize a minimizer?
- How to derive an optimization algorithm?
- How to analyze/guarantee the convergence?
- How to accelerate the convergence?
- Efficient implementation, etc.

Ready to start?