Weekly Exercise 6

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Metric (6 Points)

Exercise 1 (Metric, semi-metric, 6 Points**).** Show that the followings hold:

a) The *truncated absolute distance*, defined as $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_0^+$

$$d(x, y) = \min(K, |x - y|), \quad \text{for some } K \in \mathbb{R}^+ ,$$

is a metric.

b) The *truncated quadratic function*, defined as $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_0^+$

$$d(x, y) = \min(K, |x - y|^2), \quad \text{for some } K \in \mathbb{R}^+ ,$$

is a semi-metric.

c) The *weighted Potts-model*, defined as $d : \mathbb{N} \times \mathbb{N} \to \mathbb{R}_0^+$

$$d(\ell_1, \ell_2) = w \cdot [\![\ell_1 \neq \ell_2]\!], \quad \text{for some } w \in \mathbb{R}^+ ,$$

is a metric.

Programming (6 Points)

Exercise 2 (Binary image segmentation via maxFlow algorithm, 6 Points**).** Solve the *binary image segmentation* problem on the image in Figure 1 by applying the *Boykov–Kolmogorov maxFlow algorithm*.

Figure 1: The test image for binary image segmentation.

For binary segmentation $y_i \in \mathbb{B}$ for all $i \in \mathcal{V}$, where $\mathcal{V}$ stands for the set of pixels, furthermore 0 and 1 denote the background and the foreground, respectively. Let us consider the following energy function for $w \in \mathbb{R}^+$:

$$E(\mathbf{y};\mathbf{x}) = \sum_{i\in\mathcal{V}} E_i(y_i;x_i) + w \sum_{\{i,j\}\in\mathcal{E}} E_{ij}(y_i,y_j;x_i,x_j) \ , \tag{1}$$

where $\mathcal{E}$ includes 4-neighboring pixels and x_i consists of the RGB intensities of the pixel i.

Use the GMM models f_{bg} and f_{fg} you trained in *Exercise 4* in order to define the **unary energy functions** for all $i \in \mathcal{V}$:

$$E_i(0;x_i) = -\log(f_{\text{bg}}(x_i)) \ ,$$
$$E_i(1;x_i) = -\log(f_{\text{fg}}(x_i)) \ .$$

Moreover, simply apply the *Potts-model* in order to define the **pairwise energy functions** for all $(i,j) \in \mathcal{E}$:

$$E_{ij}(y_i,y_j;x_i,x_j) = [\![y_i \neq y_j]\!] \ .$$

Construct a flow network corresponding to the defined energy function in Eq. (1) and solve the maximum flow problem by making use of the *Boykov–Kolmogorov algorithm*. You may use the provided maxFlow implementation of the algorithm found in the supplementary material `in2329-exercise_06_supp.zip`[1].

- Choose a set of different values for w, and report what you observe.
- How are the obtained segmentation results compared to the results you obtained in *Exercise 4* (i.e. without having pairwise terms, that is regularization)?

Minimum cut and maximum flow (4 Points)

Exercise 3 (**Edmonds–Karp algorithm**, 4 Points)**.** Solve the maximum flow problem corresponding to the flow network in Figure 2 by applying the *Edmonds–Karp algorithm*. Find the minimum $s-t$ cut as well. Draw the residual network and the flow graph for each iteration.

[1] You may also find the implementation online http://pub.ist.ac.at/~vnk/software/maxflow-v3.04.src.zip

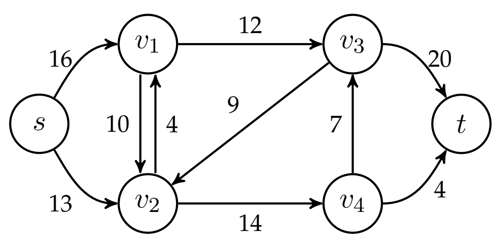

Figure 2: A flow network.