

Suggested Homework

Nonlinear Multiscale Methods for Image and Signal Analysis

Exercise 1. Consider the following problems:

- Let $J(U) = \|U\|_*$ be the nuclear norm on $\mathbb{R}^{n \times m}$, i.e. $J(U) = \sum_i \sigma_i$ where the σ_i are the classical singular values of the matrix $U \in \mathbb{R}^{n \times m}$. Determine ground states of the regularization J .
- Let K be an orthonormal matrix. Determine the singular vectors of $J(u) = \|Ku\|_1$.

Exercise 2. Visualize the subdifferential of the 1-dimensional total variation for the space being discretized by just 4 points, i.e.

$$TV(v) = \left\| \left(\begin{array}{cccc} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} \right\|_1$$

Although the corresponding subdifferential lives in four dimensions, it can be visualized as a polyhedron in three dimensions due to the deficiency of the rank of the derivative matrix K : We know that $p = K^T q$ and K^T has a column full of zeros. Thus, visualize the subdifferential by projecting along the first three principal components.

Please send me your results so I can show something it in the next lecture!