

## Weekly Exercises 9

Room: 02.09.023

Wed, 24.06.2015, 14:15-15:45

Submission deadline: Tue, 23.06.2015, 23:59 to windheus@in.tum.de

Please send in only Latex-PDF. If you have hand-written solutions, please hand them in during the lecture.

### Shape Differences

Let  $\mathcal{M}, \mathcal{N}$  be two shapes, define  $h_a^{\mathcal{M}}(f, g) = \int_{\mathcal{M}} f(p)g(p)dp$ ,  $h_a^{\mathcal{N}}(f, g) = \int_{\mathcal{N}} f(p)g(p)dp$ . Discretize the shapes' Laplace-Beltrami operators using the usual finite element method resulting in mass matrices  $M_{\mathcal{M}}, M_{\mathcal{N}}$  and stiffness matrices  $W_{\mathcal{M}}, W_{\mathcal{N}}$ , such that  $\Phi_{\mathcal{M}}, \Phi_{\mathcal{N}}$  are the eigen functions (written as columns of the matrices) and  $\Lambda_{\mathcal{M}}, \Lambda_{\mathcal{N}}$  are their eigen values (written as diagonal matrices). Recall that  $\Phi_{\mathcal{M}}$  is orthonormal with respect to  $M_{\mathcal{M}}$ , i.e.  $\Phi_{\mathcal{M}}^{\top} M_{\mathcal{M}} \Phi_{\mathcal{M}} = \text{Id}$ .

- Exercise 1** (One Point). 1. Let  $F$  be a linear transformation (functional map) between  $L^2(\mathcal{M})$  and  $L^2(\mathcal{N})$ , represented by matrix  $C$  mapping functions in the eigen basis  $\Phi_{\mathcal{M}}$  onto functions in the eigen basis  $\Phi_{\mathcal{N}}$ . The Riesz representation theorem tells us we can write  $h_a^{\mathcal{N}}(F(f), F(g)) = h_a^{\mathcal{M}}(f, D(g))$ , for any functions  $f, g \in L^2(\mathcal{M})$ , where  $D : L^2(\mathcal{M}) \rightarrow L^2(\mathcal{M})$  is a linear map. Show that  $D$  is self adjoint, i.e.  $h_a^{\mathcal{M}}(f, D(g)) = h_a^{\mathcal{M}}(D(f), g)$  for all  $f, g$ .
2. Now assume  $F$  is area preserving and  $D$  is represented by matrix  $V$  in the eigen basis  $\Phi_{\mathcal{M}}$ , show that  $V = C^{\top} C$ .
3. Define  $h_c^{\mathcal{M}}(f, g) = \int_{\mathcal{M}} \langle \nabla f(p), \nabla g(p) \rangle dp$ ,  $h_c^{\mathcal{N}}(f, g) = \int_{\mathcal{N}} \langle \nabla f(p), \nabla g(p) \rangle dp$ . Again let  $F$  be a linear transformation (functional map) between  $L^2(\mathcal{M})$  and  $L^2(\mathcal{N})$ , represented by matrix  $C$  mapping functions in the eigen basis  $\Phi_{\mathcal{M}}$  onto functions in the eigen basis  $\Phi_{\mathcal{N}}$ . Let  $f, g \in L^2(\mathcal{M})$ , such that  $f = \sum_i \phi_i \alpha_i$ ,  $g = \sum_i \phi_i \beta_i$ . Show that  $h_c^{\mathcal{N}}(F(f), F(g))$  can be represented by  $h_c^{\mathcal{M}}(F(f), F(g)) = \alpha^{\top} R \beta$ , where  $R$  is some matrix such that  $\Lambda_{\mathcal{M}} R = C^{\top} \Lambda_{\mathcal{N}} C$ .
4. Assume  $F$  is conformal, what can you say about  $R$ ?

### Programming: Shape Differences

**Exercise 2** (Two points). Download and expand the file `ex9.zip` from the lecture website. Modify the file `findAnaloguousShape.m` to implement the function as explained below. You need one data set of shapes to test implementation. Choose

and download one data set from [http://people.csail.mit.edu/drdaniel/mesh\\_animation/index.html](http://people.csail.mit.edu/drdaniel/mesh_animation/index.html) (one of the `meshes.tgz` files linked on the page). The meshes in those data sets are registered. Corresponding vertices on two meshes have the same index on both meshes. You can run the script `exercise.m` to test and visualize your solutions after you specified the correct path to the shape data set.

`function [meshIndex] = findAnaloguousShape(M1, M2, M3, meshFileNames, k)` accepts three meshes `M1`, `M2`, `M3` and a cell array of filenames `meshFileNames`. `k` is the number of eigen functions. It returns the index of the mesh that relates best to `M3` as `M2` relates to `M1`. Use the formula shown on page 22 from the lecture on *shape differences* to quantify shape analogy.