Analysis of Three-Dimensional Shapes
E. Rodolà, T. Windheuser, M. Vestner
Summer Semester 2015

Computer Vision Group
Institut für Informatik
Technische Universität München

Weekly Exercises 10

Room: 02.09.023
Wed, 08.07.2015, 14:15-15:45
Submission deadline: Tue, 07.07.2015, 23:59 to windheus@in.tum.de
Please send in only Latex-PDF. If you have hand-written solutions, please hand them in during the lecture.

Mathematics: The gradient

Exercise 1 (One bonus point)**.** Let S be a regular surface and $f : S \to \mathbb{R}$ a differentiable function.

1. Show that $df_p : T_pS \to \mathbb{R}$ is linear.

For a given point $p \in S$ find the vector $v \in T_pS$ such that...

2. ... an infinitesimal step in the direction of v increases f the most.
3. ... an infinitesimal step in the direction of v does not change the value of f.

Mathematics: Regular surfaces

Exercise 2 (One bonus point)**.** Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function. We consider its graph

$$\mathbf{x} : \mathbb{R}^2 \to \mathbb{R}^3,\ (u, v) \mapsto (u, v, g(u, v))$$

1. Show that $\mathbf{x}$ is regular and therefore a parametrized surface element.
2. Derive the first fundamental form of $\mathbf{x}$.
3. In what cases is the parametrization $\mathbf{x}$ orthogonal (conformal, isometric)?

Exercise 3 (One bonus point)**.** Let $g : I \to \mathbb{R}_{>0}$ be a smooth function. Consider

$$\mathbf{x} : I \times \mathbb{R} \to \mathbb{R}^3,\ (u, v) \mapsto (u, \cos(v)g(u), \sin(v)g(u)).$$

1. Give an interpretation of $\mathbf{x}$.
2. Show that $\mathbf{x}$ is regular and derive its first fundamental form.
3. Calculate the area $A(\mathbf{x}|_{I\times(0,2\pi)})$.
4. Calculate the surface area of a sphere with radius r.