Multiple View Geometry: Solution Exercise Sheet 1

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Part I: Theory

1. **To summarize:**

	B_1	B_2	B_3
(1) Are linearly independent	yes	yes	no
(2) Span $\mathbb{R}^3$	yes	no	yes
(3) Form a basis of $\mathbb{R}^3$	yes	no	no

More details:

B_1 : Can be shown by building a matrix and calculating the determinant: $det \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \neq 0$.

As the determinant is not zero, we know that the vectors are linearly independent. Three linear independent vectors in $\mathbb{R}^3$ span $\mathbb{R}^3$. Furthermore, three spanning vectors build a minimal set, hence, they also form a basis of $\mathbb{R}^3$.

B_2 : To span $\mathbb{R}^3$, there are at least three vectors needed.

B_3 : In $\mathbb{R}^3$, there cannot be more than three independent vectors.

2. **To summarize:**

	G_1	G_2	G_3
Form a group	no	no	yes

More details:

G_1 : Closure not given!

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{pmatrix}}_{\in G_1} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{\in G_1} = \underbrace{\begin{pmatrix} 1 & 4 & 9 \\ 2 & 0 & 12 \\ 3 & 8 & 15 \end{pmatrix}}_{\notin G_1}$$

G_2 : Neutral element not included, as $det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \neq -1$

G_3 : Yes, as can easily be shown using $det(A^{-1}) = \frac{1}{det(A)}$.

3. **To summarize:** No.

   More details (proof): Assuming the existence of four pairwise orthogonal, non-zero vectors $v_1, \ldots, v_4 \in \mathbb{R}^3$, we obtain a contradiction:

   We know that, in $\mathbb{R}^3$, there are at most 3 linearly independent vectors. Hence, we know that $\exists\ a_i$: $\sum_{i=1}^{4} a_i v_i = 0$, with at least one $a_i \neq 0$. Without loss of generality, we can assume that $a_1 = 1$, giving

   $$v_1 = a_2 v_2 + a_3 v_3 + a_4 v_4$$

   As the vectors are pairwise orthogonal, we can derive

   $$\begin{aligned} ||v_1||^2 &= \langle v_1, v_1 \rangle \\ &= \langle v_1, a_2 v_2 + a_3 v_3 + a_4 v_4 \rangle \\ &= \langle v_1, v_2 \rangle a_2 + \langle v_1, v_3 \rangle a_3 + \langle v_1, v_4 \rangle a_4 = 0, \end{aligned}$$

   which contradicts $v_1 \neq \mathbf{0}$.

Part I: Matlab

These are some possible solutions to the exercises from the Matlab introduction slides.

1. Exercise 1: There is a number of possibilities which do not require a loop, such as:

   - `out = all(all(abs(x-y) < eps))`
   - `out = sum(sum(abs(x-y) >= eps)) == 0`
   - `out = max(abs(x-y)) < eps`
   - `out = max( (x-y) .* (x-y) ) < eps*eps`

2. Exercise 2: Again there are multiple possibilities:

   - ```
     A = s:e;
     out = sum(isprime(A) .* A);
     ```
   - ```
     A = s:e;
     out = sum(A(isprime(A)));
     ```