Multiple View Geometry: Solution Exercise Sheet 6

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`http://vision.in.tum.de/teaching/ss2014/mvg2014`

Part I: Theory

1. (a) E is essential matrix $\quad\Rightarrow\quad \Sigma = \text{diag}\{\sigma, \sigma, 0\}$:

$$R_z(\pm\frac{\pi}{2})\Sigma = \begin{pmatrix} 0 & \mp 1 & 0 \\ \pm 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \mp\sigma & 0 \\ \pm\sigma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -(R_z(\pm\frac{\pi}{2})\Sigma)^\top$$

$$\begin{aligned} -\hat{T}^\top &= -(UR_z\Sigma U^\top)^\top \\ &= U(-R_z\Sigma)^\top U^\top \\ &= UR_z\Sigma U^\top \\ &= \hat{T} \end{aligned}$$

(b) i. U, V are orthogonal matrices $\Rightarrow U^\top U = Id$ and $VV^\top = Id$
R_z is a rotation matrix $\Rightarrow R_z R_z^\top = Id$

$$\begin{aligned} R^\top R &= (UR_z^\top V^\top)^\top (UR_z^\top V^\top) \\ &= VR_z U^\top U R_z^\top V^\top \\ &= VR_z R_z^\top V^\top \\ &= VV^\top \\ &= Id \end{aligned}$$

ii. U and V are special orthogonal matrices with $\det(U) = \det(V^\top) = 1$.

$$\det(R) = \det(UR_z^\top V^\top) = \underbrace{\det(U)}_{1} \cdot \underbrace{\det(R_z^\top)}_{1} \cdot \underbrace{\det(V^\top)}_{1} = 1$$

2. (a) $H = R + Tu^\top \Leftrightarrow R = H - Tu^\top$.

$$E = \hat{T}R = \hat{T}(H - Tu^\top) = \hat{T}H - \underbrace{\hat{T}T}_{=T\times T=0} u^\top = \hat{T}H$$

(b)

$$\begin{aligned} H^\top E + E^\top H &= H^\top(\hat{T}H) + (\hat{T}H)^\top H \\ &= H^\top(\hat{T}H) + H^\top \hat{T}^\top H \\ &= H^\top \hat{T} H - H^\top \hat{T} H \quad \text{(because } \hat{T} \text{ is skew-symmetric, i.e. } \hat{T}^\top = -\hat{T}\text{)} \\ &= 0 \end{aligned}$$

3. $\forall\, x_2: \quad l_1 = \{x_1 \mid x_2^\top F x_1 = 0\}$

   In particular: $e_1 \in l_1 \quad \Rightarrow \quad x_2^\top F e_1 = 0 \;\; \forall\, x_2$

   $\qquad\qquad\qquad\qquad\quad \Rightarrow \quad F e_1 = 0$

   (The camera center o_2 is in the preimage of every $x_2 \Rightarrow$ The epipol e_1 (which is the projection of o_2 to the image plane of image 1) lies on all epipolar lines l_1)

   Analogous: $e_2^\top F = 0$.