

Exercise Sheet 4

Room: 02.09.023

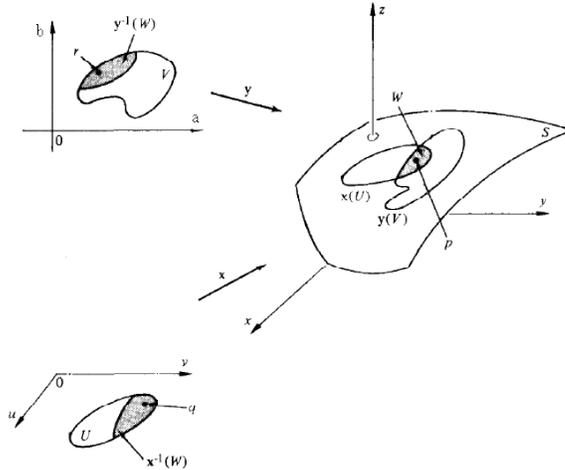
Tue, 27.05.2014, 13:45-15:15

Submission deadline: Mon, 26.05.2014, 23:59 to windheus@in.tum.de

Mathematics: More Differential Geometry

Exercise 1 (First fundamentalform). 1. Show that the first fundamental form is positive definit if and only if the parametrization is regular.

2. Let $\mathbf{x} : U \rightarrow S$ and $\mathbf{y} : V \rightarrow S$ be two parametrizations of a surface S and let $h = \mathbf{x}^{-1} \circ \mathbf{y} : V \rightarrow U$ be the diffeomorphic change of coordinates. Find an expression for the first fundamental form $g_y = d\mathbf{y}^T d\mathbf{y}$ in terms of h and \mathbf{x} . (For simplicity assume that $\mathbf{x}(U) = \mathbf{y}(V)$)



Exercise 2 (The gradient). Let S be a regular surface and $f : S \rightarrow \mathbb{R}$ a differentiable function.

1. Show that $df_p : T_p S \rightarrow \mathbb{R}$ is linear.

For a given point $p \in S$ find the vector $v \in T_p S$ such that...

2. ... an infinitesimal step in the direction of v increases f the most.
 3. ... an infinitesimal step in the direction of v does not change the value of f .

Programming: Multi-Dimensional Scaling

Exercise 3 (Two points). Download and expand the file `exercise4.zip` from the lecture website. Modify the files `mds.m` and `alignpoints.m` to implement the functions as explained below. You can run the script `exercise.m` to test and visualize your solutions.

`mds.m` Implement the multi-dimensional scaling method introduced in the lecture. The function accepts a metric given by matrix $D \in \mathbb{R}^{n \times n}$ and a dimension $m \in \mathbb{N}$. It should return the coordinates of each of the n points embedded into \mathbb{R}^m as matrix $Z \in \mathbb{R}^{n \times m}$.

Parameters `alpha`, `epsilon` and `maxI` control the gradient descent's behaviour and are the stepsize, minimum relative progress and maximum number of iterations, respectively.

`alignpoints.m` Write a function that aligns two point clouds. Given two set of points $Z_1, Z_2 \subset \mathbb{R}^m$ embedded into \mathbb{R}^m by the MDS method find two rigid transformations that are represented by $(R_1, \mathbf{t}_1), (R_2, \mathbf{t}_2) \in \mathbb{R}^{m \times m} \times \mathbb{R}^m$. The transformed point sets $\hat{Z}_1 = \{R_1(\mathbf{z} + \mathbf{t}_1) | \mathbf{z} \in Z_1\}$, $\hat{Z}_2 = \{R_2(\mathbf{z} + \mathbf{t}_2) | \mathbf{z} \in Z_2\}$ should be aligned to each other.

Translations $\mathbf{t}_1, \mathbf{t}_2$ can be found by computing the point clouds' mean. There are several ways to find good rotation matrices. We suggest to align the principal axis $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_m \in \mathbb{R}^m$ of each point cloud with the standard Euclidean axis $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_m \in \mathbb{R}^m$. Note that $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_m$ form an orthonormal basis of \mathbb{R}^m and can be computed by the matlab function `pca`.

The matlab function `alignpoints` should accept two point clouds by arguments `Z1` and `Z2` and should return the transformed points as `Zhat1` and `Zhat2`. The rotations and translations are not needed as return values.

Note that since the signs of the principal axis are not uniquely determined, the visualization code in `exercise.m` includes sign parameters that can be altered.