

## Machine Learning for Robotics and Computer Vision Summer Term 2013

### Solution Sheet 1

Topic: Introduction to Probabilistic Reasoning and Learning  
April 26, 2013

#### Exercise 1:

a)

$$p(z | x) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2} \frac{(z-x)^2}{\sigma_1^2}} = \mathcal{N}(z; x, \sigma_1) \quad (1)$$

where  $\sigma_1$  is the sensor noise.

b) The belief about the robot's state (i.e. its location) is the probability of its location given all measurements and all actions so far. So far, no measurements have been taken and the only action was "drive to the next docking station". In this very simple setup, we can assume that this action never fails. Thus, we know that the robot is located at one of the 7 docking stations, but we don't know at which docking station it is. Therefore, we can model the state variable  $x$  as a discrete random variable with values between 0 and 7, where 0 means that the robot did not find a docking station. The belief can be expressed as

$$\text{Bel}(x_0) = p(x_0 | u_0) = \begin{cases} \frac{1}{7} & \text{if } x_0 \in \{1, 2, 3, 4, 5, 6, 7\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

We could also model the uncertainty caused by the docking process and by the inaccuracy of the docking station positions. We could do this by modeling  $x$  as a continuous random variable and a mixture of 7 Gaussians for the belief:

$$\text{Bel}(x) = \frac{1}{7} \sum_{i=1}^7 \mathcal{N}(x; i, \sigma_2), \quad (3)$$

where  $\sigma_2$  models the uncertainty of the docking stations.

c)

$$p(x_0 | z_0, u_0) = \frac{p(z_0 | x_0, u_0)p(x_0 | u_0)}{\sum_{x'=1}^7 p(z_0 | x', u_0)p(x' | u_0)} \quad (4)$$

$$= \frac{\mathcal{N}(z_0; x_0, \sigma_1)p(x_0 | u_0)}{\sum_{x'=1}^7 \mathcal{N}(z_0; x', \sigma_1)p(x' | u_0)} \quad (5)$$

We can compute this with the following table:

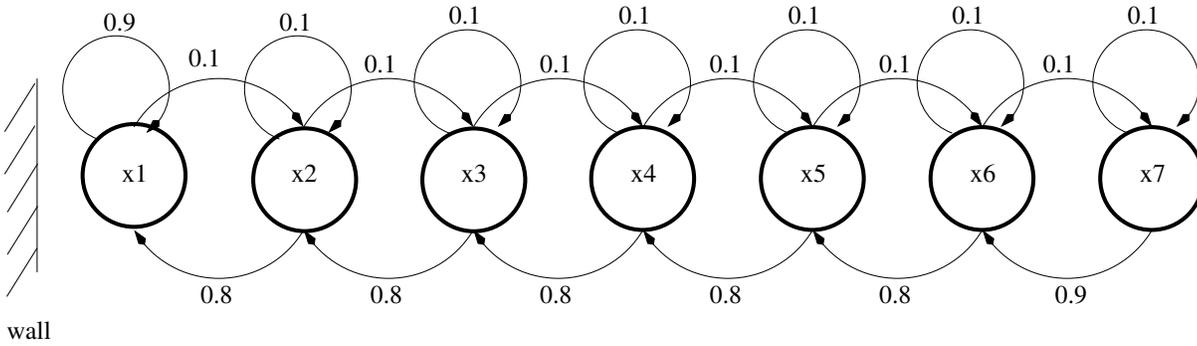


Abbildung 1: State transition diagram.

$x_0 =$	1	2	3	4	5	6	7
$\mathcal{N}(z_0; x_0, \sigma_1)$	2.641e-15	3.483e-6	0.209	0.567	6.996e-5	3.919e-13	9.97e-26
$\mathcal{N}(z_0; x_0, \sigma_2)$	0.014	0.111	0.333	0.368	0.15	0.022	0.001
$p_1(x_0   u_0, z_0)$	3.406e-15	4.491e-06	0.269	0.730989	9.021e-05	5.054e-13	1.286e-25
$p_2(x_0   u_0, z_0)$	0.0136	0.111	0.333	0.369	0.15	0.022	0.001

We can see that the robot is most likely at position  $x = 4$ .

d) State transition diagram: see Fig. 1. Posterior:

$$p(x_1 | u_0, z_0, u_1) = \sum_{x_0=1}^7 p(x_1 | u_1, x_0) p(x_0 | u_0, z_0) \quad (6)$$

This results in:

$x_0 =$	1	2	3	4	5	6	7
$p_1(x_0   u_0, z_0, u_1)$	3.593e-06	0.215	0.612	0.1	0.073	9.021e-06	5.054e-14
$p_2(x_0   u_0, z_0, u_1)$	0.101	0.28	0.339	0.19	0.07	0.018	0.002

In both cases the robot is most likely at position 3. The probability decreased, because the robot motion introduced uncertainty in the estimation.

### Exercise 2:

Here are several examples of learning algorithms:

- Mean-shift clustering: Unsupervised learning
- Perceptron algorithm: Discriminant function
- Bayes classifier: Generative model
- Conditional Random Field: Discriminative model
- AdaBoost: Discriminant function

For a detailed explanation, please see the textbook *Pattern Recognition and Machine Learning* by C.M. Bishop or the slides.