Exercise 1: Back Propagation

- Suppose we modify a single neuron in a feedforward network so that the output from the neuron is given by \( f(\sum_j w_j x_j + b) \), where \( f \) is some function other than the sigmoid.

  How should we modify the backpropagation algorithm (from chapter 2 of the above reading) in this case?

  Derivative of the activation function \( f(\cdot) \) at the corresponding neuron should be derived. During back propagation, the error should be back-propagated with \( f' \) for the corresponding neuron.

- Compute the gradient of the cost function \( C \) respect to \( w_5 \) \( \left( \frac{\partial C}{\partial w_5} \right) \) given the following network:

\[
C = \sum_{i=1}^{2} (t_i - a_i)^2 \quad \text{where} \quad t_i \text{ is the target value for the respective output neuron } o_i \quad \text{and} \quad a_i \text{ is the output of the neuron } o_i. \quad \text{Input/output of } h_1, h_2, o_1, o_2 \text{ is computed as } z_i = \sum_j w_j \cdot a_j + b_i, \quad a_i = \sigma(z_i) \text{ where } \sigma(\cdot) \text{ is any activation function.}
\]

\[
\frac{\partial C}{\partial w_5} = \frac{\partial C}{\partial a_2^1} \cdot \frac{\partial a_2^1}{\partial z_1^2} \cdot \frac{\partial z_1^2}{\partial w_5}
\]

\[
\frac{\partial C}{\partial w_5} = -2(t_1 - a_1^2) \cdot \sigma'(z_1^2) \cdot a_1^1
\]

\[
\frac{\partial C}{\partial w_5} = -2(t_1 - a_1^2) \cdot \sigma' \left( w_5 a_1^1 + w_7 a_2^1 + b_2 \right) \cdot a_1^1 \tag{1}
\]
Exercise 2: Convolutional Layer Arithmetic

Consider a very simple convolutional neural network that just consists of one convolutional layer. It has the following parameters:

- number of kernels: \(num = 64\)
- size of kernels: \(k = 3 \times 5\)
- stride: \(s = 2\)
- padding: \(p = 1\)

Assume, the input to this layer is an a batch of RGB images. There are 10 images in one batch and the images have a dimension of 123 \(\times\) 81.

In general, blobs have the canonical shape \(N \times C \times H \times W\).

a) What is the shape of the input blob to the convolutional layer? Hint: it’s a tensor with four axes.

The input blob will have the shape \(10 \times 3 \times 81 \times 123\).

b) What is the shape of the output blob of the convolutional layer?

In order to find the output blob shape, one has to consider what happens during a convolution. If there is a non-zero padding, zeros will get virtually added at the boundary of the input. So the input size increases by \(2 \times p\) where \(p\) is the padding size. Every time a kernel gets multiplied with pixels, one number for the activation map is computed. Thus, the number of times that you can ”apply” a kernel along the height/width of the input will give you the dimensions of the output. Putting all this together will give you the following formula:

\[
\tilde{x} = \frac{x + 2 \times p - k}{s} + 1
\]

where \(\tilde{x}\) is the output height or width, respectively, \(k\) is the kernel height or width and \(s\) is the stride. That means that the output blob will have the shape \(10 \times 64 \times 40 \times 62\)

Exercise 3: Adaboost

See code.